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NONLINEAR SYSTEMS

On Pulse Optimal Control of Linear Systems with Aftereffect

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Abstract—The paper deals with the problem of pulse optimal control of a linear dynamic system with aftereffect. As a functional, the degenerate quadratic functional of the most common kind is considered. The absence of control in the functional leads to the fact that the optimal control contains a pulse component. Sufficient conditions of existence of the pulse optimal control are obtained and the equations describing coefficients in the optimal control are worked out. Sufficient conditions that make it possible to integrate equations and to find the coefficients in an explicit form for the optimal control are established. A model example is considered.

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1. INTRODUCTION

The degenerate linear-quadratic optimization problems that are considered both at the finite and at the infinite time intervals are of great applied importance [1]. This is true both for the systems described by ordinary differential equations and for the systems with aftereffect. The degenerate linear-quadratic problems for the two above-mentioned classes of systems display such a feature that in the class of measurable controls these problems are unsolvable [2–5], and so to afford the existence of optimal control, it is necessary to extend the set of admissible controls, allowing for pulse controls.

In this work we consider the programmed problem of optimal stabilization with a degenerate integral quadratic functional of a rather common form. The summands with aftereffect are contained both in the right side of the system and under the integral in the functional. The problem with a similar dynamics of the system and with a structure of the functional at the finite time interval was dealt with in [4, 5]. The equations describing the coefficients in optimal control are derived. The obtained system is fairly complex, and the way of integrating it is not simple. The suggestion is made to simplify the system in the same way as in [6], defining the matrices in the functional in a special way, which makes it possible to obtain in the explicit form the coefficients in the control.

2. STATEMENT OF THE PROBLEM AND ITS REDUCTION

It is necessary to develop the control action v that affords the asymptotic stability of a zero solution of the system

$$\dot{x}(t) = Ax(t) + A_\tau x(t-\tau) + \int\limits_{-\tau}^{0} G(\theta)x(t+\theta)d\theta + B\dot{v}(t) \tag{1}$$

with the initial condition

$$x(t) = \varphi(t), \qquad t_0 - \tau \leqslant t \leqslant t_0,$$

and minimizes the functional

$$J[v(\cdot)] = \int\limits_{t_0}^{\infty} \Bigg[x^{\mathrm{T}}(t)\Phi_0 x(t) + x^{\mathrm{T}}(t) \int\limits_{-\tau}^{0} \Phi_1(\theta) x(t+\theta) d\theta$$

$$+ \int\limits_{-\tau}^{0} x^{\mathrm{T}}(t+\theta)\Phi_1^{\mathrm{T}}(\theta) d\theta\, x(t) + \int\limits_{-\tau}^{0} x^{\mathrm{T}}(t+s)\Phi_2(s) x(t+s) ds$$

$$+ \int\limits_{-\tau}^{0}\int\limits_{-\tau}^{0} x^{\mathrm{T}}(t+\theta)\Phi_3(\theta,\rho) x(t+\rho) d\theta d\rho + x^{\mathrm{T}}(t-\tau)\Phi_4 x(t-\tau) \Bigg] dt, \tag{2}$$

where Φ_0, Φ_1, Φ_2, Φ_3, Φ_4 are symmetric (apart from Φ_1 and Φ_3) continuous (by the aggregate of variables) matrix-valued functions of the dimension $n \times n$, $x(t)$, $\varphi(t)$ are vector functions of the dimension n, $v(t)$ is the vector function of the dimension m, A, A_τ are constant matrices of the dimension $n \times n$, $G(\theta)$ is the matrix-valued function of the dimension $n \times n$ with piecewise continuous elements, B is the constant matrix of the dimension $n \times m$.

The given problem is degenerate [1] and is not solvable in the class of absolutely continuous functions. To obtain an optimal solution, we will extend the problem by introducing pulse controls. Further we will assume that $v(t)$ and thus $x(t)$ are the functions of a limited variation, the derivatives of which are understood in the generalized sense [9]. The initial function $\varphi(t)$ will also be assumed the function of a limited variation.

As a solution of Eq. (1) we will take the solution of the appropriate integral equation,

$$x(t) = \varphi(t_0) + \int\limits_{t_0}^{t} A(s)x(s)ds + \int\limits_{t_0}^{t} A_\tau(s)x(s-\tau)ds$$

$$+ \int\limits_{t_0}^{t}\int\limits_{-\tau}^{0} G(\theta)x(s+\theta)d\theta ds + \int\limits_{t_0}^{t} B\, dv(s),$$

where the integrals are understood in the Riemann–Stieltjes sense.

On replacing the variable

$$y(t) = x(t) - Bv(t) \tag{3}$$

we will set up an auxiliary problem in the same way as in [4]. The function $y(t)$ will satisfy the system of equations

$$\dot y(t) = Ay(t) + A_\tau\Big(y(t-\tau) + Bv(t-\tau)\Big)$$

$$+ \int\limits_{-\tau}^{0} G(\theta)\Big(y(t+\theta) + Bv(t+\theta_1)\Big)d\theta + ABv(t). \tag{4}$$

In this system, in view of (3), we have

$$y(t-\tau) + Bv(t-\tau) = \varphi(t-\tau) \tag{5}$$

at $t \in [t_0, t_0+\tau]$. Therefore, the solution of the system (4), we consideration for (5), will coincide with the solution of the system (4) at the initial condition

$$y(t) = \varphi(t), \quad v(t) = 0, \qquad t_0 - \tau \leqslant t \leqslant t_0.$$

On replacing (3), the functional (2) takes the form

$$J^*[v(\cdot)] = \int\limits_{t_0}^{\infty} W[y, y_\tau(\cdot), v_\tau(\cdot)]\, dt, \tag{6}$$

where

$$\begin{aligned}
W[y, y_\tau(\cdot), v_\tau(\cdot)] &= \Big(y(t) + Bv(t)\Big)^{\mathrm{T}} \Phi_0 \Big(y(t) + Bv(t)\Big) \\
&+ \Big(y(t) + Bv(t)\Big)^{\mathrm{T}} \int\limits_{-\tau}^{0} \Phi_1(\theta) \Big(y(t+\theta) + Bv(t+\theta)\Big) d\theta \\
&+ \int\limits_{-\tau}^{0} \Big(y(t+\theta) + Bv(t+\theta)\Big)^{\mathrm{T}} \Phi_1^{\mathrm{T}}(\theta) d\theta \Big(y(t) + Bv(t)\Big) \\
&+ \int\limits_{-\tau}^{0} \Big(y(t+s) + Bv(t+s)\Big)^{\mathrm{T}} \Phi_2(s) \Big(y(t+s) + Bv(t+s)\Big) ds \\
&+ \int\limits_{-\tau}^{0} \int\limits_{-\tau}^{0} \Big(y(t+\theta) + Bv(t+\theta)\Big)^{\mathrm{T}} \Phi_3(\theta, \rho) \Big(y(t+\rho) + Bv(t+\rho)\Big) d\theta d\rho \\
&+ \Big(y(t-\tau) + Bv(t-\tau)\Big)^{\mathrm{T}} \Phi_4 \Big(y(t-\tau) + Bv(t-\tau)\Big).
\end{aligned}$$

3. SOLUTION OF THE AUXILIARY PROBLEM

The method of dynamic programming is used for the solution of an auxiliary problem. The control action will be sought in the form

$$v(t) = W_0 y(t) + \int\limits_{-\tau}^{0} W_1(\theta) y(t+\theta)\, d\theta + \int\limits_{-\tau}^{0} W_2(\theta_1) v(t+\theta_1)\, d\theta_1. \tag{7}$$

Note that if the control action takes the form (7), the trajectory of a controllable system with the control (7) will satisfy the system

$$\begin{aligned}
\dot{y}(t) &= Ay(t) + A_\tau \left(y(t-\tau) + Bv(t-\tau)\right) \\
&+ \int\limits_{-\tau}^{0} G(\theta) \Big(y(t+\theta) + Bv(t+\theta_1)\Big)\, d\theta + ABv(t), \\
\dot{v}(t) &= W_0 \Bigg[Ay(t) + A_\tau \Big(y(t-\tau) + Bv(t-\tau)\Big) \\
&+ \int\limits_{-\tau}^{0} G(\theta) \Big(y(t+\theta) + Bv(t+\theta)\Big)\, d\theta + ABv(t) \Bigg] \\
&+ W_1(0) y(t) - W_1(-\tau) y(t-\tau) - \int\limits_{-\tau}^{0} \frac{dW_1(\theta)}{d\theta} y(t+\theta)\, d\theta \\
&+ W_2(0) v(t) - W_2(-\tau) v(t-\tau) - \int\limits_{-\tau}^{0} \frac{dW_2(\theta)}{d\theta} v(t+\theta)\, d\theta.
\end{aligned}$$

For an optimal value of the functional (6), at the position of $\{y, y_\tau(\cdot), v_\tau(\cdot)\}$ (Bellman functional), we will introduce the designation

$$I(y(t), y_\tau(\cdot), v_\tau(\cdot)) = \min_{v(\cdot)} J^*[v(\cdot)].$$

The Bellman functional will be sought in the form

$$I(y(t), y_\tau(\cdot), v_\tau(\cdot)) = y^{\mathrm{T}}(t)Py(t) + 2y^{\mathrm{T}}(t)\int_{-\tau}^{0} Q(s)y(s)\,ds$$
$$+ \int_{-\tau}^{0}\int_{-\tau}^{0} y^{\mathrm{T}}(s)R(s,\rho)y(\rho)\,dsd\rho + 2y^{\mathrm{T}}(t)\int_{-\tau}^{0} P_1(\theta_1)v(\theta_1)d\theta_1$$
$$+ 2\int_{-\tau}^{0}\int_{-\tau}^{0} y^{\mathrm{T}}(\theta)P_2(\theta,\theta_1)y(\theta_1)\,d\theta d\theta_1 + \int_{-\tau}^{0}\int_{-\tau}^{0} v^t(\theta)P_3(\theta,\theta_1)v(\theta_1)\,d\theta d\theta_1$$
$$+ \int_{-\tau}^{0} y^{\mathrm{T}}(\theta)P_4(\theta)y(\theta)\,d\theta + 2y^{\mathrm{T}}(t-\tau)\int_{-\tau}^{0} P_5(p)v(p)\,dp + \int_{-\tau}^{0} v^{\mathrm{T}}(r)P_6(r)v(r)\,dr.$$

Theorem 1. *Let for $t \in [t_0, \infty)$ the following conditions be fulfilled:*
(a) *the matrices P, $Q(s)$, $R(s,\theta)$, $P_4(s)$ are the solution of the system*

$$Q(0) + Q^{\mathrm{T}}(0) + P^{\mathrm{T}}A + A^{\mathrm{T}}P + P_4(0) + \Phi_0 = F_0^{\mathrm{T}}H^{-1}F_0, \tag{8}$$
$$-\frac{d}{d\theta}Q(\theta) + R(0,\theta) + PG(\theta) + A^{\mathrm{T}}Q(\theta) + \Phi_1(\theta) = F_0^{\mathrm{T}}H^{-1}F_1(\theta), \tag{9}$$
$$-\left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\rho}\right)R(\theta,\rho) + G^{\mathrm{T}}(\theta)Q(\rho) + Q^{\mathrm{T}}(\theta)G(\rho) + \Phi_3(\theta,\rho) = F_1^{\mathrm{T}}(\theta)H^{-1}F_1(\rho), \tag{10}$$
$$-\frac{d}{ds}P_4(s) + \Phi_2(s) = 0 \tag{11}$$

under boundary conditions

$$A_\tau^{\mathrm{T}}P = Q^{\mathrm{T}}(-\tau), \quad A_\tau^{\mathrm{T}}Q(\theta) + Q^{\mathrm{T}}(\theta)A_\tau = R^{\mathrm{T}}(\theta,-\tau) + R(-\tau,\theta), \quad \Phi_4 = P_4(-\tau) \tag{12}$$

for $-\tau \leqslant \theta, \rho, s \leqslant 0$; for the functions $P_1(s)$, $P_2(s,\theta)$, $P_3(s,\theta)$, $P_5(s)$, $P_6(s)$ the following presentations are valid:

$$P_1(\theta_1) = Q(\theta_1)B, \quad P_3(\theta_1,\theta_2) = B^{\mathrm{T}}R(\theta_1,\theta_2)B,$$
$$P_2(\theta,\theta_1) = R(\theta,\theta_1)B, \quad P_5(p) = P_4(p)B, \quad P_6(r) = B^{\mathrm{T}}P_4(r)B;$$

(b) *the matrix $B^{\mathrm{T}}\Phi_0 B + P_6(0)$ is positive definite;*
(c) *the matrix*

$$\begin{pmatrix} \frac{1}{\tau}\Phi_0 & \Phi_1(\theta) \\ \Phi_1^{\mathrm{T}}(\theta) & \Phi_2(\theta) \end{pmatrix}$$

is positive semidefinite for $\theta \in [-\tau, 0]$;

(d) *the matrix* $\Phi_3(\theta, \rho)$ *has the structure*

$$\Phi_3(\theta, \rho) = \widetilde{\Phi}_3^{\mathrm{T}}(\theta)\, \widetilde{\Phi}_3(\rho),$$

where $\widetilde{\Phi}_3(\cdot)$ *is some* $n \times n$ *matrix with piecewise continuous elements;*

(e) *the matrix* Φ_4 *is positive semidefinite;*

(f) *the Bellman functional* $I(y(t), y_\tau(\cdot), v_\tau(\cdot))$ *is positive definite.*

Then the auxiliary problem of optimal stabilization has a solution and the optimal stabilizing control has the form

$$v(t) = W_0 y(t) + \int_{-\tau}^{0} W_1(\theta) y(t+\theta)\, d\theta + \int_{-\tau}^{0} W_2(\theta_1) v(t+\theta_1)\, d\theta_1, \tag{13}$$

where

$$W_0 = -H^{-1} F_0, \quad W_1(\theta) = -H^{-1} F_1(\theta), \quad W_2(\theta_1) = -H^{-1} F_2(\theta_1),$$
$$H = B^{\mathrm{T}} \Phi_0 B + P_6(0),$$
$$F_0 = P_1^{\mathrm{T}}(0) + B^{\mathrm{T}} A^{\mathrm{T}} P + P_5^{\mathrm{T}}(0) + B^{\mathrm{T}} \Phi_0,$$
$$F_1(\theta) = B^{\mathrm{T}} A^{\mathrm{T}} Q(\theta) + P_2^{\mathrm{T}}(\theta, 0) + B^{\mathrm{T}} \Phi_1(\theta),$$
$$F_2(\theta_1) = B^{\mathrm{T}} A^{\mathrm{T}} P_1(\theta_1) + P_3^{\mathrm{T}}(\theta_1, 0) + B^{\mathrm{T}} \Phi_1(\theta_1) B = F_1(\theta_1) B.$$

The proof of Theorem 1 is given in the Appendix.

Remark. Let

$$\Phi_1(\theta) = F_0^{\mathrm{T}} H^{-1} F_1(\theta) - R(0, \theta) - PG(\theta),$$
$$\Phi_3(\theta, \rho) = F_1^{\mathrm{T}}(\theta) H^{-1} F_1(\rho) - G^{\mathrm{T}}(\theta) Q(\rho) - Q^{\mathrm{T}}(\theta) G(\rho).$$

Then Eq. (9) takes the form

$$\frac{d}{d\theta} Q(\theta) = A^{\mathrm{T}} Q(\theta),$$

and Eq. (10) becomes

$$-\left(\frac{\partial}{\partial \theta} + \frac{\partial}{\partial \rho}\right) R(\theta, \rho) = 0.$$

Equations (8) and (11) will remain as before, and also the boundary conditions (12) for them.

4. DEVELOPMENT OF OPTIMAL PROGRAMMED CONTROL FOR THE INITIAL PROBLEM

For the optimal control of the initial problem the following theorem is valid:

Theorem 2. *The optimal programmed control in the initial problem has the form*

$$\dot{v}(t) = \Delta v(t_0, \varphi(\cdot)) \delta(t - t_0) + \dot{v}_r(t),$$

where

$$\dot{v}_r(t) = \Big(W_0 A + W_1(0)\Big) x(t) + \Big(W_0 A_\tau - W_1(-\tau)\Big) x(t-\tau)$$
$$+ \int_{-\tau}^{0} \left(G(0) - \frac{dW_1(\theta)}{d\theta}\right) x(t+\theta)\, d\theta \tag{14}$$

is the integrable function,

$$\Delta v(t_0, \varphi(\cdot)) = W_0 \varphi(t_0) + \int_{-\tau}^{0} W_1(\theta)\varphi(t_0 + \theta)\, d\theta. \tag{15}$$

It is easy to make sure that Theorem 2 is valid by differentiating the control $v(t)$ from (13) in the generalized sense [9], considering that $v(t) = 0$ at $t < t_0$.

5. EXAMPLE

Consider the controllable system

$$\dot{x}(t) = x(t) - \frac{1}{5}x(t-\tau) + \dot{v}(t). \tag{16}$$

The initial function $\varphi(t)$ is preset by the condition $\varphi(t) = 1$ at $t \in [-1; 0]$. Note that in the absence of control the system (16) is unstable.

As a functional (2) we will take

$$J[v(\cdot)] = \int_0^{\infty} \left[x^2(t) + x(t)\int_{-1}^{0} x(t+\theta)d\theta + \int_{-1}^{0} x^2(t+\theta)d\theta + \frac{1}{12}\left(\int_{-1}^{0} x(t+\theta)d\theta\right)^2 + x^2(t-1)\right] dt.$$

A trajectory jump caused by the pulse action $\Delta v(t_0, \varphi(\cdot))\delta(t)$ at the initial instant of time, according (15), is preset by the quantity $-\frac{7}{6}$. Therefore, under the action of the initial pulse $\Delta v(t_0, \varphi(\cdot))\delta(t)$ the phase point moves from the position $x = 1$ to the position $x = -\frac{1}{6}$. The motion is then carried out under the action of the regular control (14) $\dot{v}_r(t) = -\frac{7}{6}x(t) + \frac{11}{30}x(t-1)$. The modeling results are shown in figure.

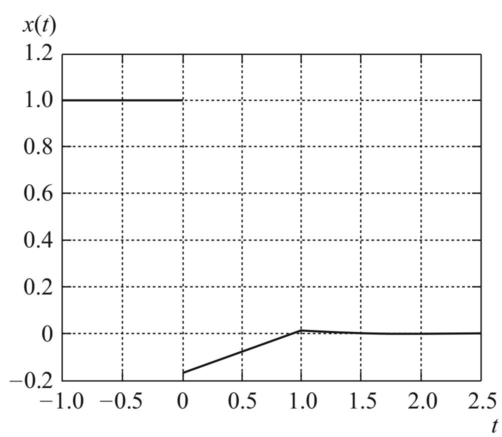

Figure.

For the numerical solution of the equation use was made of the algorithm of the Runge–Kutta–Felberg method of the fourth order with a variable step and the interpolation of previous history by means of degenerate cubic splines. The realization of the given algorithm is a part of the package of Time-Delay System Toolbox programs, which is described in [10].

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APPENDIX

Proof of Theorem 1. For an optimal value of the functional (6) at the position $\{y, y_\tau(\cdot), v_\tau(\cdot)\}$ we will introduce the designation

$$I^0(y(t), y_\tau(\cdot), v_\tau(\cdot)) = \min_{v(\cdot)} J^*[v(\cdot)].$$

The Bellman equation for the problem considered will have the form

$$\min_{v(t)} \left\{ \frac{dI^0(y(t), y_\tau(\cdot), v_\tau(\cdot))}{dt} + W[y(t), y_\tau(\cdot), v_\tau(\cdot)] \right\} = 0.$$

Using the method from [7, 8], it is possible to establish that the matrices P, $Q(s)$, $R(s,\theta)$, $P_1(s)$, $P_2(s,\theta)$, $P_3(s,\theta)$, $P_4(s)$, $P_5(s)$, $P_6(s)$, defining the optimal control and an optimal value of the functional, must satisfy the system

$$Q(0) + Q^{\mathrm{T}}(0) + P^{\mathrm{T}}A + A^{\mathrm{T}}P + P_4(0) + \Phi_0 = F_0^{\mathrm{T}} H^{-1} F_0,$$

$$-\frac{d}{d\theta} Q(\theta) + R(0,\theta) + PG(\theta) + A^{\mathrm{T}} Q(\theta) + \Phi_1(\theta) = F_0^{\mathrm{T}} H^{-1} F_1(\theta), \tag{A.1}$$

$$-\left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\rho}\right) R(\theta,\rho) + G^{\mathrm{T}}(\theta) Q(\rho) + Q^{\mathrm{T}}(\theta) G(\rho) + \Phi_3(\theta,\rho) = F_1^{\mathrm{T}}(\theta) H^{-1} F_1(\rho), \tag{A.2}$$

$$-\frac{d}{d\theta_1} P_1(\theta_1) + P_2(0,\theta_1) + A^{\mathrm{T}} P_1(\theta_1) + PG(\theta_1)B + \Phi_1(\theta_1)B = F_0^{\mathrm{T}} H^{-1} F_2(\theta_1), \tag{A.3}$$

$$-\left(\frac{\partial}{\partial\theta} + \frac{\partial}{\partial\theta_1}\right) P_2(\theta,\theta_1) + Q^{\mathrm{T}}(\theta) G(\theta_1) B + G^{\mathrm{T}}(\theta) P_1(\theta_1) + \Phi_3(\theta,\theta_1)B = F_1^{\mathrm{T}}(\theta) H^{-1} F_2(\theta_1),$$

$$-\left(\frac{\partial}{\partial\theta_1} + \frac{\partial}{\partial\theta_2}\right) P_3(\theta_1,\theta_2) + B^{\mathrm{T}} G^{\mathrm{T}}(\theta_1) P_1(\theta_2) + B^{\mathrm{T}} \Phi_3(\theta_1,\theta_2) B = F_2^{\mathrm{T}}(\theta_1) H^{-1} F_2(\theta_2), \tag{A.4}$$

$$-\frac{d}{ds} P_4(s) + \Phi_2(s) = 0,$$

$$-\frac{d}{dp} P_5(p) + \Phi_2(p) B = 0,$$

$$-\frac{d}{dr} P_6(r) + B^{\mathrm{T}} \Phi_2(r) B = 0$$

under boundary conditions

$$\begin{gathered}
PA_\tau B = P_1(-\tau), \quad B^{\mathrm{T}} A_\tau^{\mathrm{T}} Q(\theta) = P_2^{\mathrm{T}}(\theta, -\tau), \quad A_\tau^{\mathrm{T}} P = Q^{\mathrm{T}}(-\tau), \\
A_\tau^{\mathrm{T}} Q(\theta) + Q^{\mathrm{T}}(\theta) A_\tau = R^{\mathrm{T}}(\theta, -\tau) + R(-\tau, \theta), \\
A_\tau^{\mathrm{T}} P_1(\theta_1) = P_2(-\tau, \theta_1), \\
B^{\mathrm{T}} A_\tau^{\mathrm{T}} P_1(\theta_1) + P_1^{\mathrm{T}}(\theta_1) A_\tau B = P_3^{\mathrm{T}}(\theta_1, -\tau) + P_3(-\tau, \theta_1), \\
\Phi_4 = P_4(-\tau), \quad \Phi_4 B = P_5(-\tau), \\
B^{\mathrm{T}} \Phi_4(t) B = P_6(-\tau) \qquad \text{(A.5)}
\end{gathered}$$

for $-\tau \leqslant \theta, \theta_1, \theta_2, \rho, p, r, s \leqslant 0$.

Comparing Eqs. (A.1) and (A.3) with consideration for the boundary conditions (A.5), we make sure of the validity of the equality $P_1(\theta_1) = Q(\theta_1)B$. From the comparison of Eqs. (A.2) and (A.4) we verify the validity of the equality $P_3(\theta_1, \theta_2) = B^{\mathrm{T}} R(\theta_1, \theta_2) B$.

Condition (b) appears because the control in the Bellman equation is sought as a result of the fulfilment of the minimization operation in $v(t)$.

Condition (f) ensures stabilizability of the optimal trajectory, while conditions (b), (d) and (e) are necessary for the fulfilment of condition (f).

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